# ME 308 MACHINE ELEMENTS 2

**DEPARTMENT OF MECHANICAL ENGINEERING**

# UNIVERSITY OF GAZIANTEP

# POGO STICK SPRING DESIGN

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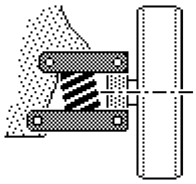
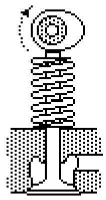
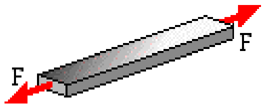
## INTRODUCTION

## SPRINGS

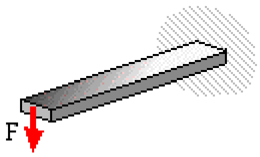


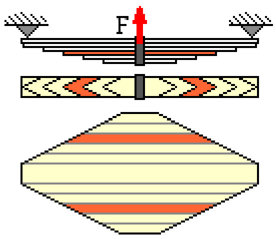


Springs are unlike other machine/structure components in that they undergo significant deformation when loaded - their compliance enables them to store readily recoverable mechanical energy. In a vehicle suspension, when the wheel meets an obstacle, the springing allows movement of the wheel over the obstacle and thereafter returns the wheel to its normal position. Another common duty is in cam follower return - rather than complicate the cam to provide positive drive in both directions, positive drive is provided in one sense only, and the spring is used to return the follower to its original position. Springs are common also in force- displacement transducers, eg. in weighing scales, where an easily discerned displacement is a measure of a change in force.




The simplest spring is the tension bar. This is an efficient energy store since all its elements are stressed identically, but its deformation is small if it is made of metal. Bicycle wheel spokes are the only common applications which come to mind.



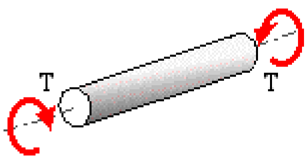




Beams form the essence of many springs. The deflection $\delta$ of the load F on the end of a cantilever can be appreciable - it depends upon the cantilever's geometry and elastic modulus, as predicted by elementary beam theory. Unlike the constant cross- section beam, the *leaf spring* shown on the right is stressed almost constantly along its length because the linear increase of bending moment from either simple support is matched by the beam's widening - not by its deepening, as longitudinal shear cannot be transmitted between the leaves.

The shortcoming of most metal springs is that they rely on either bending or torsion to obtain significant deformations; the stress therefore varies throughout the material so that the material does not all contribute uniformly to energy storage. The wire of a *helical compression spring* - such as shown on the left - is loaded mainly in torsion and is therefore usually of circular cross- section. This type of spring is the most common and we shall focus on it. The *(ex)tension spring* is similar to the compression spring however it requires special ends to permit application of the load - these ends assume many forms but they are all potential sources of weakness not present in compression springs. Rigorous duties thus usually call for compression rather than tension springs.

A tension spring can be wound with initial pre-load so that it deforms only after the load reaches a certain minimum value. Springs which are loaded both in tension and in compression are rare and restricted to light duty.

A *Belleville washer* or 'disc spring' is a washer manufactured to a conical surface as shown by the cross- sections in this photograph. Belleville washers are characterised by short axial lengths and relatively small deformations, and are often used in stacks as illustrated. Their geometry can be engineered to produce highly non-linear characteristics which may display negative stiffness.




All the abovementioned springs are essentially translatory in that forces and linear deflections are involved. Rotary springs involve torque and angular deflection. The simplest of these is the torsion bar in which loading is pure torque; its analysis is based upon the simple torsion equation. Torsion bars are stiff compared to other forms of rotary spring, however they do have many practical applications such as in vehicle suspensions.

Torsion springs which are more compliant than the torsion bar include the clock- or *spiral torsion spring (left)* and the *helical torsion spring (right)*. These rely on bending for their action, as a simple free body will quickly demonstrate. The helical torsion spring is similar to the helical tension spring in requiring specially formed ends to transmit the load.

The *constant force spring* is not unlike a self- retracting tape measure and is used where large relative displacements are required - the spring motors used in sliding door closers is one application. There exists also a large variety of non-metallic springs often applied to shock absorption and based on rubber blocks loaded in shear. Springs utilising gas compressibility also find some use.

## GIVEN PARAMETERS FOR POGO STICK SPRING DESIGN

*Mass of User: 90 kg*
*Surface Condition: Unpeened*
*Reliability: 90%*
*Working Deflection: 87 mm*
*Minimum Factor of Safety: 1,4*
*dA: 65 mm*
*dB: 50 mm*
*hB : 15 mm*
*Rod Diameter: 15 mm*

## ASTM A-232 and A-231 Materials Properties and Wire Diameters

| Material | Material Properties | Maximum Working Temp. | Ultimate Tensile Range, ksi | Modulus of Elasticity, psi $10^6$ | Approx. Design % of Ultimate Tensile (torsional) | Common Sizes, in |
|---|---|---|---|---|---|---|
| Chrome Vanadium<br>ASTM A-231<br>Valve Quality:<br>ASTM A-232<br>AMS 6450 | Cold drawn. Good for shock loads and medium elevated temperature applications. Susceptible to hydrogen embrittlement when plated. | 425°F | 325/190 | (E) 30<br>(G) 11.5 | 45% | .043" to .500" |

## SIZE CHART

| Wire Dia., in. | Wire Dia., in. | Wire Dia., in. | Wire Dia., in. | Wire Dia., in. | Wire Dia., in. | Wire Dia., in. | Wire Dia., in. |
|---|---|---|---|---|---|---|---|
| .043 | .060 | .091 | .102 | .125 | .207 | .312 | .406 |
| .046 | .062 | .092 | .105 | .135 | .225 | .331 | .437 |
| .054 | .072 | .095 | .113 | .162 | .262 | .343 | .468 |
| .059 | .080 | .099 | .120 | .187 | .283 | .375 | |
| **Valve Quality** | | | | | | | |
| **Wire Diameter, in** | | | | | | | |
| .059 | .095 | .135 | .162 | .207 | .283 | .343 | |
| .085 | .109 | .142 | .177 | .243 | .306 | | |
| .090 | .125 | .148 | .192 | .262 | .331 | | |

## TENSILE STRENGTH CHART

| Diameter, inches | Tensile Strength, ksi[A] | | Reduction of Area, minimum, % | Diameter, inches | Tensile Strength, ksi[A] | | Reduction of Area, minimum, % |
|---|---|---|---|---|---|---|---|
| | minimum | maximum | | | minimum | maximum | |
| .020 | 300 | 325 | C | .162 | 225 | 245 | 40 |
| .032 | 290 | 315 | C | .192 | 220 | 240 | 40 |
| .041 | 280 | 305 | C | .244 | 210 | 230 | 40 |
| .054 | 270 | 295 | C | .283 | 205 | 225 | 40 |
| .062 | 265 | 290 | C | .312 | 203 | 223 | 40 |
| .080 | 255 | 275 | C | .375 | 200 | 220 | 40 |
| .105 | 245 | 265 | 45 | .438 | 195 | 215 | 40 |
| .135 | 235 | 255 | 45 | .500 | 190 | 210 | 40 |

**OUTER SPRING DESIGN**
*Let's say outer spring takes x% of the total load and x=60, 60% assume total force.*

Mass of User: 90 kg g=10m/s2

Fmax=90*10=900 N

For Fmin calculation assume mass of user 45 kg.

Fmin=45*10=450 N

Fmax outer= Fmax*60/100 ➔ 900*60/100=540 N

Fmin outer= Fmin*60/100 ➔ 450*60/100= 270 N

dA=65mm
dB=50mm

Do outer < dA Do outer < 65
Di outer < dB Di outer < 50

**FIRST ASSUMPTION;**

*$D_{o,outer}$ = 63mm*
*d= 5mm*

*$D_{o,outer}$ < dA*
*63 < 65 mm ➔ It is OK!*

*D= $D_{o,outer}$ - d= 63-5=58 mm*

*$D_{i,outer}$ = D-d= 58-5=53 mm*

*$D_{i,outer}$ > 50 ➔ 53>50 ➔ It is OK!*

*Spring Index Factor;*

$4 \leq C \leq 12$

$C = \frac{D}{d} = \frac{58}{5} = 11{,}6$ ➔ *It is OK!*

$Ks = 1 + \frac{0{,}5}{C} = 1{,}0431$

**1- STATIC DESIGN FOR OUTER SPRING**

$$\tau_{max} = \frac{K_s \times 8 \times F_{max} \times D}{\pi \times d^3}$$

$$\tau_{max} = \frac{1{,}0431 \times 8 \times 540 \times 58}{\pi \times 5^3}$$

$$= 665{,}5478\ Mpa$$

*The spring material is chosen Chrome-Vanadium from Table 10.2.*

A=2000 *m*=0,167

$$S_{ut} = \frac{A}{d^m} = \frac{2000}{5^{0.167}} = 1528{,}6287\ Mpa$$

$$S_y = 0{,}75 * S_{ut} = 1146{,}4715\ Mpa$$

$$S_{sy} = 0{,}57 * S_y = 661{,}5141\ Mpa$$

$$\eta = \frac{S_{sy}}{\tau_{max}} = \frac{661{,}5141}{665{,}5478} = 0{,}993939$$

$0{,}993939 < 1{,}4$ ➔ *It is too little.*

*As a result, the true assumption is shown following table.*

| *Do,outer* *mm* | *d* *mm* | *D* mm | *Di,outer* mm | *C* | *Ks* | $\tau_{max}$ Mpa | $S_{ut}$ Mpa | $S_y$ Mpa | $S_{sy}$ Mpa | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| *63* | *5* | *58* | *53* | *11,6* | *1,0431* | *665,5478* | *1528,6287* | *1146,4715* | *661,6760* | *0,993939* |
| *63* | *6* | *57* | *51* | *9,5* | *1,0526* | *381,9719* | *1482,7869* | *1112,0902* | *641,6760* | *1,679904* |

## 2- FATIGUE DESIGN FOR OUTER SPRING

$$Fa = \frac{Fmax - Fmin}{2} = \frac{900 - 540}{2} = 225\ N$$

$$Fa, max = \frac{Fa * 60}{100} = \frac{225 * 60}{100} = 135\ N$$

$$\tau_a = \frac{K_s * 8 * F_{max} * D}{\pi * d^3} = \tau_{max} = \frac{1,0526 * 8 * 135 * 57}{\pi * 6^3} = 95,493\ Mpa$$

$$S_{se} = kc * kd * ke * S_{se}{}'$$

$S_{se} = 310\ Mpa$ ➔ *For unpeened is corrected for ka and kb.*

$kc = 0,897$ ➔ *For reliability 90% (from Table 7.7)*

$kd = 1$ ➔ *Assume temperature* $\leq 350°C$

$$K_{wahl} = \frac{4C - 1}{4C - 4} + \frac{0,615}{C} = \frac{4 * 9,5 - 1}{4 * 9,5 - 4} + \frac{0,615}{9,5} = 1,15297$$

$$KC = \frac{K_{wahl}}{ks} = \frac{1,15297}{1,0526} = 1,09532$$

$$ke = \frac{1}{KC} = \frac{1}{1,09532} = 0,91297$$

$$S_{se} = 0,897 * 1 * 0,91297 * 310$$

$$S_{se} = 253,87\ Mpa$$

$\eta_f = \frac{S_{se}}{\tau a} = \frac{253,87}{95,493} = 2,65852$ ➔ *It means that spring goes to infinite life.*

$$k = \frac{\Delta F}{\Delta y} = \frac{Fmax - Fmin}{Yworking} = \frac{540 - 270}{87} = 3,1034$$

$N_a = \frac{d^4 * G}{8 * D^3 * k} = \frac{6^4 * 79,3 * 10^3}{8 * 57^3 * 3,1034} = 22,35 \cong 22,5$ *coils*

Nd=2 ➔ *For both tip of the spring is squared and ground.*

Nt = 22,5+2 = 24,5 *coils*

$$L_{solid} = Nt * d$$

$$L_{solid} = 24,5 * 6 = 147\ mm$$

$$Y_{working} = 87\ mm$$

$$Y_{clash} = 0,1 * Y_{working} = 0,1 * 87 = 8,7\ mm$$

$$Y_{initial} = \frac{Fmin}{k} = \frac{270}{3,1034} = 87\ mm$$

$$L_f = L_s + Y_w + Y_{clash} + L_{initial}$$

$$L_f = 147 + 87 + 8,7 + 87$$

$$L_f = 329,7$$

***Tip:*** *There is no risk of buckling. Because there is a rod inside of the springs. This method is used for prevent buckling.*

## 3- PERMANENT SET:

$$\tau_{solid} < S_y$$

$$\tau_{solid} = \frac{K_s * 8 * F_{solid} * D}{\pi * d^3}$$

$$F_{solid} = k * (L_f - L_s) = 3,1034 * (329,7 - 147) = 567\ N$$

$$\tau_{solid} = \frac{1,0526 * 8 * 567 * 57}{\pi * 6^3} = 401,0704\ Mpa$$

$\eta_{solid} = \frac{S_{sy}}{\tau_{solid}} = \frac{661,5141}{401,0704} = 1,5999$ ➔ *It is OK!*

***Note;*** *No permanent set occurs when compressed solid.*

## 4- CRITICAL FREQUENCY TO PREVENT RESONANCE

$\rho = 7800\ kg/m^3$

$A = \frac{\pi d^2}{4} = \frac{\pi*(6*10^{-3})^2}{4} = 2,82 * 10^{-5}\ m^2$

$L = \pi * D * Nt = \pi * 57 * 24,5 * 10^{-3} = 4,387\ m$

$Ms = \rho * A * L = 7800 * 2,82 * 10^{-5} * 4,387 = 0,967\ kg$

$$F_n = \frac{1}{2}\sqrt{\frac{k * 1000}{Ms}}$$

$$F_n = \frac{1}{2}\sqrt{\frac{3,1034*1000}{0,967}} = 28,3173 \quad \frac{rad}{sec}$$

*It is converted to hertz ;*

$$F_n = \frac{28,3173}{2 * \boldsymbol{\pi}}$$

$$F_n = 4,506\ Hertz$$

### NATURAL FREQUENCY;

$$F_{natural} = \frac{4,506}{15} = 0,3\ Hertz$$

***Tip;*** *Frequency calculation is important. Because, If frequency of the spring is same as naturel frequency, it will get resonance. So, calculated frequency must be divided by 15.*

### *INNER SPRING CALCULATION*

**OUTER SPRING DESIGN**

*Let's say outer spring takes x% of the total load and x=40, 40% assume total force.*

Mass of User: 90 kg
g=10m/s2

Fmax=90*10=900 N

*For Fmin calculation assume mass of user 45 kg.*

*Fmin*=45*10=450 N

Fmax outer= Fmax*40/100 ➔ 900*40/100=360 N

Fmin outer= Fmin*40/100 ➔ 450*40/100= 180 N

dB=50 mm
Rod=15 mm

| | |
|---|---|
| Do inner < dB | Do inner < 50 |
| Di outer > Rod | Di inner < 15 |

**FIRST ASSUMPTION;**

*$D_{o,inner}$ = 48mm*
*d= 4mm*

*$D_{o,inner}$ < dB*
*48 < 50 mm ➔ It is OK!*

*D= $D_{o,inner}$ - d= 48-4=44 mm*

*$D_{i,inner}$ = D-d= 44-4=40 mm*

*$D_{i,outer}$ < 50 ➔ 40 < 50 ➔ It is OK!*

*Spring Index Factor;*

$4 \leq C \leq 12$

$C = \frac{D}{d} = \frac{44}{4} = 11$ ➔ *It is OK!*

$Ks = 1 + \frac{0,5}{C} = 1,0455$

## 1- STATIC DESIGN FOR OUTER SPRING

$$\tau_{max} = \frac{K_s \times 8 \times F_{max} \times D}{\pi \times d^3}$$

$$\tau_{max} = \frac{1,0455 \times 8 \times 360 \times 44}{\pi \times 4^3}$$

$$= 658,9015\ Mpa$$

*The spring material is chosen Chrome-Vanadium from Table 10.2.*

A=2000 $m$=0,167

$$S_{ut} = \frac{A}{d^m} = \frac{2000}{4^{0.167}} = 1586,6677\ Mpa$$

$$S_y = 0,75 * S_{ut} = 1190,0008\ Mpa$$

$$S_{sy} = 0,57 * S_y = 686,6304\ Mpa$$

$$\eta = \frac{S_{sy}}{\tau_{max}} = \frac{686,6304}{658,9015} = 0,993939$$

1,042084 < 1,4 ➔ *It is too little.*

*As a result, the true assumption is shown following table.*

| *Assumptions* | *Do,inner* mm | *d* mm | *D* mm | *Di,inner* mm | *C* | *Ks* | $\tau_{max}$ Mpa | $S_{ut}$ Mpa | $S_y$ Mpa | $S_{sy}$ Mpa | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *First Assumption* | *48* | *4* | *44* | *40* | *11* | *1,0455* | *658,9015* | *1586,6677* | *1190,0008* | *686,6304* | *1,042084* |
| *Second Assumption* | *47,9* | *4,5** | *43,4* | *38,9* | *9,644* | *1,0518* | *459,2465* | *1555,7632* | *1166,8224* | *673,2565* | *1,466003* |

* *This wire diameter is chosen from catalogue of Chrome-Vanadium ASTM A-232.*

## 2- FATIGUE DESIGN FOR OUTER SPRING

$$Fa = \frac{Fmax - Fmin}{2} = \frac{900 - 540}{2} = 225\ N$$

$$Fa, max = \frac{Fa * 40}{100} = \frac{225 * 40}{100} = 90\ N$$

$$\tau_a = \frac{K_s * 8 * F_{max} * D}{\pi * d^3}$$

$$\tau_a = \frac{1,0526 * 8 * 90 * 43,9}{\pi * 4,5^3} = 114,812\ Mpa$$

$$S_{se} = kc * kd * ke * S_{se}'$$

$S_{se} = 310\ Mpa$ ➔ *For unpeened is corrected for ka and kb.*

$kc = 0,897$ ➔ *For reliability 90% (from Table 7.7)*

$kd = 1$ ➔ *Assume temperature* $\leq 350°C$

$$K_{wahl} = \frac{4C - 1}{4C - 4} + \frac{0,615}{C} = \frac{4 * 9,64 - 1}{4 * 9,64 - 4} + \frac{0,615}{9,64}$$

$$K_{wahl} = 1,15053$$

$$KC = \frac{K_{wahl}}{ks} = \frac{1,15053}{1,0518} = 1,09382$$

$$ke = \frac{1}{KC} = \frac{1}{1,09382} = 0,91423$$

$$S_{se} = 0,897 * 1 * 0,91423 * 310$$

$$S_{se} = 254,219\ Mpa$$

$\eta_f = \frac{S_{se}}{\tau a} = \frac{254,219}{114,812} = 2,212423$ ➔ *It means that spring goes to infinite life.*

$$k = \frac{\Delta F}{\Delta y} = \frac{Fmax - Fmin}{Yworking} = \frac{360 - 180}{87} = 2,06897$$

$N_a = \frac{d^4 * G}{8 * D^3 * k} = \frac{4,5^4 * 79,3 * 10^3}{8 * 43,4^3 * 2,06897} = 24,0332 \cong 24$ *coils*

Nd=2 ➔ *For both tip of the spring is squared and ground.*

Nt = 24+2 = 26 *coils*

$$L_{solid} = Nt * d$$

$$L_{solid} = 26 * 4,5 = 117\ mm$$

$$Y_{working} = 87\ mm$$

$Y_{clash} = 0{,}1 * Y_{working} = 0{,}1 * 87 = 8{,}7\ mm$

$Y_{initial} = \dfrac{Fmin}{k} = \dfrac{180}{2{,}06897} = 87\ mm$

$L_f = L_s + Y_w + Y_{clash} + L_{initial}$

$L_f = 117 + 87 + 8{,}7 + 87$

$L_f = 229{,}7mm$

***Tip:*** *There is no risk of buckling. Because there is a rod inside of the springs. This method is used for prevent buckling.*

### 3- PERMANENT SET:

$\tau_{solid} < S_y$

$\tau_{solid} = \dfrac{K_s * 8 * F_{solid} * D}{\pi * d^3}$

$F_{solid} = k * (L_f - L_s)$
$= 2{,}06897 * (229{,}7 - 117) = 378\ N$

$\tau_{solid} = \dfrac{1{,}0518 * 8 * 378 * 43{,}4}{\pi * 4{,}5^3}$
$\tau_{solid} = 482{,}2088\ Mpa$

$\eta_{solid} = \frac{S_{sy}}{\tau_{solid}} = \frac{673{,}2565}{482{,}2088} = 1{,}4$ ➔ *It is OK!*

***Note;*** *No permanent set occurs when compressed solid.*

### 4- CRITICAL FREQUENCY TO PREVENT RESONANCE

$\rho = 7800\ kg/m^3$

$A = \frac{\pi d^2}{4} = \dfrac{\pi*(6*10^{-3})^2}{4}$
$A = 1{,}5904 * 10^{-5}\ m^2$

$L = \pi * D * Nt = \pi * 43{,}4 * 26 * 10^{-3}$
$= 3{,}544\ m$

$Ms = \rho * A * L = 7800 * 1{,}5904 * 10^{-5} * 3{,}544$
$Ms = 0{,}439\ kg$

$F_n = \dfrac{1}{2}\sqrt{\dfrac{k * 1000}{Ms}}$

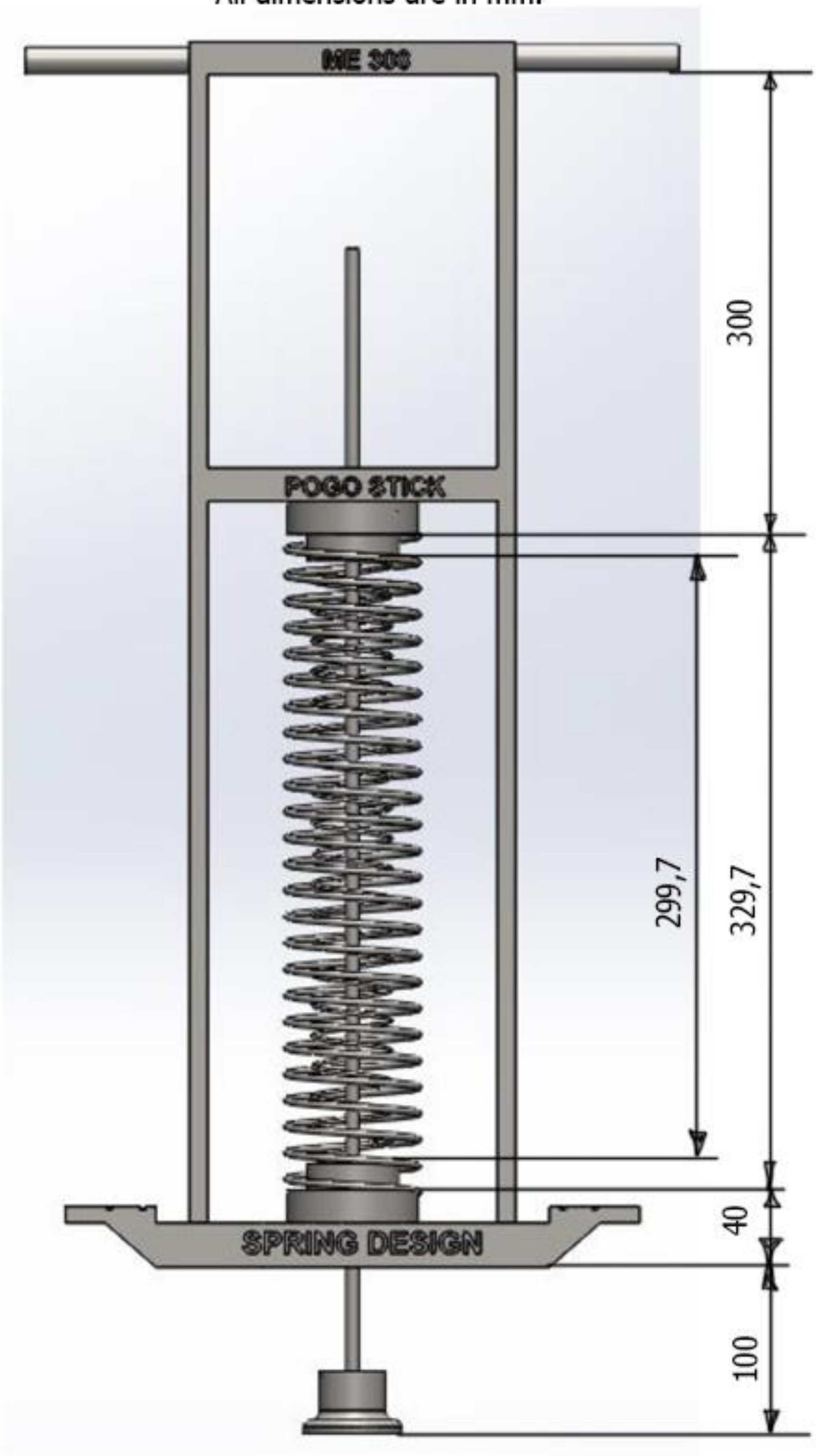
$F_n = \frac{1}{2}\sqrt{\frac{2{,}06897*1000}{0{,}967}} = 34{,}2953 \quad \frac{rad}{sec}$

*It is coverted to hertz;* ➔ $\frac{34{,}2953}{2*\pi} = 5{,}458\ Hertz$

***Natural Frequency;***

$F_{natural} = \dfrac{5{,}458}{15} = 0{,}3638\ Hertz$

All dimensions are in mm.



**CONCLUSION**

In this project, we calculated a pogo stick spring by given parameters. Necessary safety factors were determined at the same time spring length and diameters, too. As a result, pogos stick dimensions are determined.